ERRATUM - WILSON CONJECTURE FOR OMEGA-CATEGORICAL LIE ALGEBRAS, THE CASE 4-ENGEL CHARACTERISTIC 3

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There is a mistake in the proof of Theorem 4.11 in the published version. The statement of the theorem is nonetheless still true, as we will see in this note. I am very thankful to Iker de las Heras for pointing out the mistake. Theorem 4.11 is the following statement:

Theorem 0.1. *Any 5-strong Lie algebra over a field of size ≥ 4 satisfy the identity*

$$[x, y, y, y, y, z, z, z, z] = [x, z, z, z, z, y, y, y, y].$$

This is the case $n = 5$ of a conjecture of mine on n-strong Lie algebras: if the field has size large enough, they satisfy the identity $x^{n-1}y^{n-1} = (-1)^{n-1}y^{n-1}x^{n-1}$.

Let L be a 5-strong Lie algebra over a field of size ≥ 5, let $a, b, c_1, c_2, c_3, c_4 \in L$ and λ a scalar. From $I(a + \lambda b)^5 = 0$ we have that

$$(a + \lambda b)c_1(a + \lambda b)c_2(a + \lambda b)c_3(a + \lambda b)c_4(a + \lambda b) = 0.$$

The mistake occcurs when computing the right hand side, where the three terms in red were missing

$(a + \lambda b)c_1(a + \lambda b)c_2(a + \lambda b)c_3(a + \lambda b)c_4(a + \lambda b) =$

$\quad ac_1ac_2ac_3ac_4a$

$\quad + \lambda \overbrace{(ac_1ac_2ac_3ac_4b + ac_1ac_2ac_3bc_4a + ac_1ac_2bc_3ac_4a + ac_1bc_2ac_3ac_4a + bc_1ac_2ac_3ac_4a)}^{=:A}$

$\quad + \lambda^2 \overbrace{(ac_1ac_2ac_3bc_4b + ac_1ac_2bc_3ac_4b + ac_1bc_2ac_3ac_4b + bc_1ac_2ac_3ac_4b + bc_1ac_2ac_3bc_4a + bc_1ac_2bc_3ac_4a}^{=:B}$

$\qquad + bc_1bc_2ac_3ac_4a + {\color{red}ac_1bc_2bc_3ac_4a + ac_1bc_2ac_3bc_4a + ac_1ac_2bc_3bc_4a})$

$\quad + \lambda^3 \overbrace{(bc_1bc_2bc_3ac_4a + bc_1bc_2ac_3bc_4a + bc_1ac_2bc_3bc_4a + ac_1bc_2bc_3bc_4a + ac_1bc_2bc_3ac_4b + ac_1bc_2ac_3bc_4b}^{=:C}$

$\qquad + ac_1ac_2bc_3bc_4b + {\color{red}bc1ac2ac3bc4b + bc1ac2bc3ac4b + bc1bc2ac3ac4b})$

$\quad + \lambda^4 \overbrace{(bc_1bc_2bc_3bc_4a + bc_1bc_2bc_3ac_4b + bc_1bc_2ac_3bc_4b + bc_1ac_2bc_3bc_4b + ac_1bc_2bc_3bc_4b)}^{=:D}$

$\quad + \lambda^5 bc_1bc_2bc_3bc_4b$

By hypotheses, $ac_1ac_2ac_3ac_4a = bc_1bc_2bc_3bc_4b = 0$ and because the underlying field is large enough, we obtain $A = B = C = D = 0$. Note that the proof that $a^4b^4 = 0$ follows from those equations without the red terms is correct, however, the identity $a^4b^4 = 0$ does note follow from the correct set of equations above.

Now, if $a, b \in L$, let $A(a, b)$ be the associative subalgebra of $\mathrm{End}(L)$ generated by $\mathrm{ad}_a, \mathrm{ad}_b$. Because $I(a)$ and $I(b)$ are nilpotent of class < 5, every word in a, b of length ≥ 9 vanishes, hence $A(a, b)$ is nilpotent of class at most $8 = 2 \times 4$. More generally, the associative algebra generated by the adjoint of s elements of a k-strong Lie algebra is nilpotent of class $\leq sk$.

Now in order to check that $a^4b^4 = b^4a^4$, we use the GAP package ModIsom. We can compute a 2-generated associative algebra with generators a, b, submitted to the relations R given by $A = B = C = D = 0$ for all values of c_i ranging in the set $\{1, a, b, ab\}$ ($c_i = 1$ means that we omit c_i from the equation). Let us call it $A = \mathbb{F}_p[X, Y]/R$. We then compute a nilpotent quotient of this algebra and it stabilizes at nilpotency class 8 as an associative algebra of dimension 130 over $\mathbb{F}_p$ for $p = 2, 3, 5, 7$. We check that in A we have $a^4b^4 = b^4a^4$. As we know that $A(a, b)$ if of nilpotency class ≤ 8 and that there is the map

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$A \to A(a, b)$ we know that we have $a^4 b^4 = b^4 a^4$ also hold in $A(a, b)$, so the theorem holds. Note that the ring A is not of nilexponent 5 but seems to be of nilexponent 6.

References

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